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COVARIANT CANONICAL QUANTIZATION OF THE GREEN-SCHWARZ SUPERSTRING

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Introducing a new type of $D = 10$ harmonic superspace with two generations of harmonic coordinates, we reduce the Green-Schwarz (GS) superstring to a system whose constraints are Lorentz covariant and functionally independent. These features allow us to impose Lorentz-covariant gauge fixing conditions for the reparametrization and the fermionic κ-invariances. The resulting Q_{BRST} corresponds to the finite-dimensional Lie algebra of the remaining purely harmonic constraints. The super-Poincaré symmetry acts in a manifestly Lorentz-covariant form and is apparently anomaly free.

1. Introduction and motivation

Realization of the importance of manifest space-time supersymmetry for the superstring theory (anomaly cancellation, finiteness, vanishing cosmological constant etc.; see ref. [1] for a detailed discussion and a long list of references) attracted a lot of interest in the Green-Schwarz (GS) formulation of the superstrings [2–6].

In spite of a few years of intensive efforts, progress towards the super-Poincaré covariant quantization of the GS superstrings was obstructed until now by the following serious problems.

The original GS action defines a constrained system which contains a complicated mixture of first and second class constraints. These constraints cannot be separated in a Lorentz-covariant way [7].

The covariant separation proposed in refs. [3, 5] leads in fact to sets of *functionally dependent* constraints (*reducible sets* of constraints, according to the terminology of Batalin-Fradkin-Vilkovisky (BFV) [8–10]).

It was pointed out in ref. [11] that the application of the correct BFV procedure [9, 10] to these reducible constraints would either break the Lorentz invariance (as it

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is the case in ref. [12]*), or would force the level of reducibility [9] to be ∞ (which is intractable).

Ref. [11] proposed a new formalism for consistent covariant canonical quantization of the $N = 2$ Brink-Schwarz (BS) superparticle [14, 15] envisaging generalization to the case of the GS superstring. The main ingredient was the introduction of additional pure gauge degrees of freedom – harmonic variables carrying Lorentz-spinor indices. These were crucial in order to obtain an irreducible set of constraints without destroying manifest Lorentz invariance and without altering the physical content of the model.

An unpleasant feature of this formalism was the fact that the BRST charge Q_{BRST} became rank two.

Also, some obscure points persisted:

(a) The geometrical meaning of the spinor harmonics.

(b) The origin (and the reality properties) of the first class constraints and the way they substituted the old second class ones.

In ref. [13] the following successes were achieved in applying the $D = 10$ harmonic superspace formalism to the $N = 2$ BS superparticle:

(a) A significantly simpler and geometrically more meaningful realization of the $D = 10$, $N = 2$ harmonic superspace was introduced. The bosonic harmonic coordinates carried Lorentz-spinor as well as Lorentz-vector indices. In particular the rank of the new Q_{BRST} was one.

(b) The BFV-BRST covariant second quantization [16] of the $N = 2$ BS superparticle produced an off-shell unconstrained superfield action for the linearized $D = 10$, type II B supergravity.

(c) We elucidated the mechanism of disentangling the second class constraints and the methods of their treatment.

In the present paper we extend the formalism of ref. [13] to the GS superstring. First, we use the Lorentz spinor/vector harmonics to separate, in a Lorentz-covariant and functionally independent form, the first- and second-class constraints of the GS superstring. Second, we impose, with the help of the harmonics, Lorentz-covariant gauge fixing conditions for the local fermionic κ-invariance and for the reparametrization invariance. Third, we introduce a new, second generation of harmonic variables with a simple geometrical meaning of their own. They enable us to disentangle the resulting Dirac brackets for the remaining canonical superstring coordinates and to recognize them as sets of canonically conjugated pairs.

We end up with a covariant superstring action (eq. (4.19)) whose sole constraints are the remaining purely harmonic first-class constraints forming a finite-dimensional Lie-algebra (which is apparently anomaly free).

* Lorentz invariance is explicitly broken in the formalism of ref. [12] by the introduction of two constant light-like vectors which are not dynamical degrees of freedom.

The main feature of this action is that the super-Poincaré symmetry acts in a manifestly Lorentz-covariant way and is apparently anomaly free.

The plan of the paper is as follows. In sect. 2 we briefly recall the definition and the basic properties of the Lorentz-spinor/vector harmonics [13] corresponding to the coset space SO(1,9)/(SO(8) × SO(1,1)). Then, we introduce the second generation of harmonics realizing the coset space SO(8)/(SU(4) × U(1)). In sect. 3 we supplement the GS superstring action with the action for the (pure-gauge) harmonic degrees of freedom. They enable a Lorentz-covariant irreducible disentangling of the first class constraints from the second class constraints. In sect. 4 we fix in a Lorentz-covariant way the local fermionic and reparametrization invariances. In sect. 5 we discuss the modification of the supersymmetry transformations necessary to preserve the Lorentz-covariant gauge fixing conditions of sect. 4. We find the explicit form of the super-Poincaré generators. The algebra is apparently anomaly free. Appendix A contains our spinor conventions. Appendix B sketches the proof that the harmonic variables are pure gauge degrees of freedom.

The main results of the paper are concentrated in the formulae (4.15)–(4.22) which contain the canonical variables, the action in the hamiltonian form, the constraints and their BRST charge. Since the κ-and reparametrization-invariances are fixed, Q_{BRST} contains only information about the harmonic constraints.

2. Two-generation harmonic superspace in $D = 10$

The formalism of ref. [11, 13] as well as the present formalism rely heavily on the concept of harmonic superspace which was first proposed in ref. [17] as a basic ingredient of the unconstrained, off-shell superfield formulation of $N = 2, 3$ matter-, gauge- and supergravity theories in $D = 4$.

The harmonic superspace of ref. [17] is parametrized by $(x^{\mu}, \theta_{A\alpha}, u)$ where $(x^{\mu}, \theta_{A\alpha})$ are the ordinary (extended) superspace coordinates and the additional bosonic coordinates u belong to a coset space G/H. Here G denotes an appropriate global symmetry group of the supersymmetric theory and H is an appropriate subgroup of G. The harmonics u serve as "bridges" converting G-covariant indices into H-covariant ones, preserving at the same time the G-symmetry. In the case $D = 4$, one has $\mathrm{G} = \mathrm{SU}(N)$ (the automorphism group of N-extended supersymmetry) and $\mathrm{H} = \mathrm{U}(1)^{N-1}$ for $N = 2, 3$ [17].

Let us now recall the definition of the Lorentz-spinor/vector harmonic superspace of ref. [13] which combines the nice features of the Lorentz-spinor harmonic space of ref. [11] with the previously introduced covariant "light-cone" harmonic superspace [18].

The corresponding harmonic variables, which we now call first generation harmonics, consist of the following objects:

(i) $v_{\alpha}^{+1/2}$-two $D = 10$ (left-handed) Majorana-Weyl (MW) spinors,

(ii) u_μ^a-eight ($a = 1,\dots,8$) $D = 10$ Lorentz vectors, which satisfy the constraints*:

$$\left[v_\alpha^{-\frac{1}{2}}(\sigma^\mu)^{\alpha\beta}v_\beta^{+\frac{1}{2}}\right]\left[v_\gamma^{-\frac{1}{2}}(\sigma_\mu)^{\gamma\delta}v_\delta^{-\frac{1}{2}}\right] = -1,$$

$$\left[v_\alpha^{\mp\frac{1}{2}}(\sigma^\mu)^{\alpha\beta}v_\beta^{\pm\frac{1}{2}}\right]u_\mu^a = 0,$$

$$u_\mu^a u^{b\mu} = C^{ab}. \tag{2.1}$$

In the last line of (2.1), C^{ab} denotes the $D = 8$ charge-conjugation matrix. The group $\mathrm{H} = \mathrm{SO}(8) \times \mathrm{SO}(1,1)$ acts as an internal group of *local* rotations on u_μ^a, $v_\alpha^{\pm\frac{1}{2}}$ where u_μ^a, transform as SO(8) (s)-spinors whereas $v_\alpha^{\pm\frac{1}{2}}$ carry $\pm\frac{1}{2}$ charge under SO(1,1).

Because of the celebrated $D = 10$ Fierz identity (cf. e.g. ref. [1]):

$$(\sigma_\mu)^{\alpha\beta}(\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\gamma}(\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha}(\sigma^\mu)^{\beta\delta} = 0 \tag{2.2}$$

the composite Lorentz vectors:

$$u_\mu^\pm = v_\alpha^{\pm\frac{1}{2}}(\sigma_\mu)^{\alpha\beta}v_\beta^{\pm\frac{1}{2}} \tag{2.3}$$

are identically light-like and, therefore, the first generation harmonics realize through (2.1), (2.3) the coset space SO(1,9)/(SO(8) × SO(1,1)).

Let us now introduce the second generation of harmonics w_a^k, $\overline{w}_a^k$ which realizes the coset space SO(8)/(SU(4) × U(1)). They are defined as follows:

$$w_a^k w^{la} = \overline{w}_a^k \overline{w}^{la} = 0,$$

$$w_a^k \overline{w}^{la} = C^{kl}, \tag{2.4}$$

or, equivalently:

$$C_{kl}\left(w_a^k \overline{w}_b^l + w_b^k \overline{w}_a^l\right) = C_{ab}. \tag{2.5}$$

Here $C^{kl} = C^{lk}$ denotes the $D = 6$ charge conjugation matrix (recall that locally SU(4) ≈ SO(6)). C^{kl} is used to raise and lower SU(4) spinor indices (cf. appendix A). The two groups, $\mathrm{H} = \mathrm{SO}(8) \times \mathrm{SO}(1,1)$ and $\mathrm{K} = \mathrm{SU}(4) \times \mathrm{U}(1)$, act as internal groups of local rotations on w_a^k, w_a^k which transform as (8,0) with respect to H and as $(4, +\frac{1}{2})$, $(\bar{4}, -\frac{1}{2})$ with respect to K.

Henceforth, we shall use the shorthand notations:

$$u_\mu^\pm \qquad \text{as in eq. (2.3)},$$

$$\sigma^a \equiv \sigma^\mu u_\mu^a,$$

$$\sigma^\pm \equiv \sigma^\mu\left(v^{\pm\frac{1}{2}}\sigma_\mu v^{\pm\frac{1}{2}}\right). \tag{2.6}$$

* All spinor notations and conventions are explained in appendix A.

The following harmonic differential operators preserve the harmonic constraints (2.1), (2.4) and will play an important role in the sequel:

$$D^{ab} = u_\mu^a \frac{\partial}{\partial u_{\mu b}} - u_\mu^b \frac{\partial}{\partial u_{\mu a}} + w^{ak}\frac{\partial}{\partial w_b^k} + \bar{w}^{a\dot{k}}\frac{\partial}{\partial \bar{w}_b^{\dot{k}}} - w^{bk}\frac{\partial}{\partial w_a^k} - \bar{w}^{b\dot{k}}\frac{\partial}{\partial \bar{w}_a^{\dot{k}}}, \tag{2.7}$$

$$D^{-a} = u_\mu^+ \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2} v^{+\frac{1}{2}}\sigma^+\sigma^a \frac{\partial}{\partial v^{-\frac{1}{2}}},$$

$$D^{-+} = \frac{1}{2}\left(v_\alpha^{+\frac{1}{2}} \frac{\partial}{\partial v_\alpha^{+\frac{1}{2}}} - v_\alpha^{-\frac{1}{2}} \frac{\partial}{\partial v_\alpha^{-\frac{1}{2}}}\right),$$

$$E^{IJ} = w_a^k \left(\frac{1}{2}\rho^{IJ}\right)_k^{\ l} \frac{\partial}{\partial w_a^l} + \bar{w}_a^{\dot{k}} \left(\frac{1}{2}\tilde{\rho}^{IJ}\right)_{\dot{k}}^{\ \dot{l}} \frac{\partial}{\partial \bar{w}_a^{\dot{l}}},$$

$$E^{+-} = \frac{1}{2}\left(w_a^k \frac{\partial}{\partial w_a^k} - \bar{w}_a^{\dot{k}} \frac{\partial}{\partial \bar{w}_a^{\dot{k}}}\right),$$

$$E^{+I} = \frac{1}{\sqrt{2}} w_a^k (\rho^I)_k^{\ \dot{l}} \frac{\partial}{\partial \bar{w}_a^{\dot{l}}}. \tag{2.8}$$

We have used the notations:

$$\rho^{IJ} = \tfrac{1}{2}(\rho^I\tilde{\rho}^J - \rho^J\tilde{\rho}^I),$$

$$\tilde{\rho}^{IJ} = \tfrac{1}{2}(\tilde{\rho}^I\rho^J - \tilde{\rho}^J\rho^I),$$

where $\rho^I, \tilde{\rho}^I (I = 1,\dots,6)$ are the $D = 6$ σ-matrices (cf. appendix A).

The D's and E's form the algebras (only the nonzero commutators are listed):

$$[D^{ab}, D^{cd}] = C^{bc}D^{ad} - C^{ac}D^{bd} + C^{ad}D^{bc} - C^{bd}D^{ac},$$

$$[D^{ab}, D^{+c}] = C^{bc}D^{+a} - C^{ac}D^{+b},$$

$$[D^{-+}, D^{-a}] = D^{+a}, \tag{2.9}$$

$$[E^{IJ}, E^{LM}] = \delta^{IL}E^{JM} + \delta^{JK}E^{IL} - \delta^{JL}E^{IK} - \delta^{IK}E^{JL},$$

$$[E^{IJ}, E^{+L}] = \delta^{JL}E^{+I} - \delta^{IL}E^{+J},$$

$$[E^{+-}, E^{+I}] = E^{+I}, \tag{2.10}$$

from which one immediately recognizes:

(i) D^{ab}, D^{+-} as generators of SO(8) × SO(1, 1) and D^{+a} as the half of the coset generators corresponding to SO(1, 9)/(SO(8) × SO(1, 1));

(ii) E^{IJ}, E^{+-} as generators of SU(4) × U(1) and E^{+I} as the half of the coset generators corresponding to SO(8)/(SU(4) × U(1));

In appendix B we sketch the proof of the following important property.

Let

$$\Phi = \Phi\Big[X^{\mu}(\cdot), \theta_{A\alpha}(\cdot); u^{a}_{\mu}, v^{\pm\frac{1}{2}}_{a}, w^{k}_{a}, w^{\dot{k}}_{a} \Big]$$

be an arbitrary superstring field, i.e. a functional of the superstring coordinates $x^{\mu}(\xi), \theta_{A\alpha}(\xi)$ and a function of the harmonic coordinates defined in (2.1), (2.4).

If Φ satisfies the harmonic differential equations:

$$\begin{aligned} D^{ab}\Phi = D^{+-}\Phi = D^{+a}\Phi = 0\,, \\ E^{IJ}\Phi = E^{+-}\Phi = E^{+I}\Phi = 0\,, \end{aligned} \tag{2.11}$$

then Φ is in fact independent of the harmonic coordinates.

This property is the statement (on the first quantized level) about the pure gauge nature of u, v, w (cf. next section). It ensures the complete equivalence between our "harmonized" GS action and the standard GS superstring action.

The reason why our harmonics play an essential role in the subsequent construction lies in the fact that we perform a suitable transformation mixing in a nontrivial way X^{μ}, $\theta_{A\alpha}$ with u, v, w so that we can solve covariantly the irreducible covariantly disentangled set of first- and second-class superstring constraints (cf. sects. 3 and 4).

3. Reformulation of the GS superstring

The standard form of the GS superstring action reads:

$$S_{GS} = \int d\tau d\xi \sqrt{-g}\,\Big\{ -\tfrac{1}{2} g^{mn} \Pi^{\mu}_{m} \Pi_{n\mu} + i\varepsilon^{mn} \Pi^{\mu}_{m} \Big[\theta_1 \sigma_{\mu} \partial_n \theta_1 - \theta_2 \sigma_{\mu} \partial_n \theta_2 \Big] \\ - \varepsilon^{mn} \big(\theta_1 \sigma_{\mu} \partial_m \theta_1 \big) \big(\theta_2 \sigma^{\mu} \partial_n \theta_2 \big) \Big\}\,, \tag{3.1}$$

where

$$\Pi^{\mu}_{m} \equiv \partial_m X^{\mu} + i \sum_{A=1,2} \big(\theta_A \sigma^{\mu} \partial_m \theta_A \big)\,. \tag{3.2}$$

In eq. (3.1), $g_{mn}(\tau, \xi)$ $(m, n = 0, 1)$ is the 2D world-sheet metric and $X^{\mu}(\tau, \xi)$, $\theta_A(\tau, \xi)$, $A = 1, 2$ are the superstring coordinates.

In the case of the IIB theory the θ's are two $D=10$ MW spinors $\theta_A = \theta_{A\alpha}$ with equal (left-handed) chiralities. In the case IIA they have opposite chiralities: $\theta_1 = \theta_{1\alpha}$, $\theta_2 = \theta_2^{\alpha}(= -C^{\alpha\dot\beta}\theta_{2\dot\beta})$. We always use $D=10$ σ-matrices with undotted indices (cf. appendix A).

The bosonic constraints issuing from the action (3.1) are:

$$P_g \approx 0 \tag{3.3}$$

(the Weyl invariance which from now on is gauge fixed by $g_{mn} - e^{\Phi}\delta_{mn} = 0$),

$$T_A \equiv \Pi_A^2 + 4i(-1)^A D_A \theta_A' \approx 0 \tag{3.4}$$

(the generators of the reparametrization invariance). We used in (3.4) the quantity:

$$\Pi_A^{\mu} \equiv \mathscr{P}^{\mu} + (-1)^A\left[X'^{\mu} + 2i\theta_A\sigma^{\mu}\theta_A\right], \tag{3.5}$$

where $\mathscr{P}^{\mu}$ denotes the canonically conjugated momentum to X^{μ}.

The fermionic constraints arising from (3.1) read:

$$D_A = -ip_{\theta A} - \left[\mathscr{P}^{\mu} + (-1)^A\left(X'^{\mu} + i\theta_A\sigma^{\mu}\theta'_A\right)\right]\left(\sigma_{\mu}\theta_A\right) \approx 0, \tag{3.6}$$

where $p_{\theta A}$ denotes the canonically conjugated momentum to θ_A.

The algebra of the constraints (3.4), (3.6) is:

$$\{T_A(\xi), T_B(\eta)\}_{\mathrm{PB}} = 8(-1)^A\delta_{AB}\left[T_A(\xi)\delta'(\xi-\eta) + \tfrac{1}{2}T_A'(\xi)\delta(\xi-\eta)\right], \tag{3.7}$$

$$\{D_A(\xi), D_B(\eta)\}_{\mathrm{PB}} = 2i\delta_{AB}\delta(\xi-\eta)\not{\Pi}_A(\xi), \tag{3.8}$$

$$\{D_A(\xi), T_B(\eta)\}_{\mathrm{PB}} = 4(-1)^A\delta_{AB}D_A(\eta)\delta'(\xi-\eta). \tag{3.9}$$

It is clear from (3.4), (3.7)–(3.9) that the fermionic constraints (3.6) form a mixture of 16 first-class and 16 second-class constraints.

The D's (3.6) contain a first class part which expresses the fact that part of the θ_A's are pure gauge degrees of freedom. These first class constraints are the generators of the κ-symmetries [2,4], and their action is similar with the way in which covariant supersymmetric derivatives eliminate reducible superfield components.

The second class part of the fermionic constraints expresses the fact that part of the θ_A's are their own canonical conjugates (as it is the case for ordinary Majorana fermions).

The main problem was until now the functionally independent covariant separation of the first and second class constraints from D_A (eq. (3.6)).

The solution, as described in the present paper and already accomplished in the case of the $D=10$ superparticles [13,19], is achieved through the introduction of additional pure gauge harmonic degrees of freedom.

The new action, which we write in the hamiltonian form, reads:

$$S = S_{\mathrm{GS}} + S^{(\mathrm{I})}_{\mathrm{harmonic}} + S^{(\mathrm{II})}_{\mathrm{harmonic}}, \tag{3.10}$$

$$S_{\mathrm{GS}} = \int \mathrm{d}\tau \int_{-\pi}^{\pi} \mathrm{d}\xi \left[\mathscr{P}_\mu \, \partial_\tau X^\mu + \sum_A p_{\theta A} \, \partial_\tau \theta_A - \sum_A \Lambda_A T_A - \sum_A M_A D_A \right], \tag{3.11}$$

$$S^{(\mathrm{I})}_{\mathrm{harmonic}} = \int \mathrm{d}\tau \left[p^a_{u\mu} \, \partial_\tau u^\mu_a + p_v^{\mp\frac{1}{2}\alpha} \, \partial_\tau v_\alpha^{\pm\frac{1}{2}} - \Lambda_{ab} d^{ab} - \Lambda^{+-} d^{-+} - \Lambda^-_a d^{+a} \right], \tag{3.12}$$

$$S^{(\mathrm{II})}_{\mathrm{harmonic}} = \int \mathrm{d}\tau \left[p^a_{wk} \, \partial_\tau w^k_a + p^a_{\bar{w}k} \, \partial_\tau \bar{w}^k_a - \lambda^{IJ} e^{IJ} - \lambda^{+-} e^{+-} - \lambda^{+I} e^{+I} \right]. \tag{3.13}$$

The constraints d^{ab}, d^{-+}, d^{+a} in (3.12) and e^{IJ}, e^{+-}, e^{+I} in (3.13) denote the classical counterpart of D^{ab}, D^{-+}, D^{+a} in (2.7) and E^{IJ}, E^{+-}, E^{+I} in (2.8), respectively. All constraints are first class, except D^α_A which is a mixture of first- and second-class.

Because of the kinematical constraints (2.1), (2.4) on the variables u^a_μ, $v^{\pm\frac{1}{2}}_\alpha$, w^k_a, $\bar{w}^k_a$ defining our harmonic superspace, their conjugate momenta are similarly kinematically constrained*:

$$\begin{gathered} p_u^{\mu(a} u_\mu^{b)} = 0\,, \\ p^a_{u_\mu}\left(v^{\mp\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}} \right) = 0\,, \\ v_\alpha^{+\frac{1}{2}} p_v^{-\frac{1}{2}\alpha} + v_\alpha^{-\frac{1}{2}} p_v^{+\frac{1}{2}\alpha} = 0\,, \end{gathered} \tag{3.14}$$

$$\begin{gathered} p^a_{wk} \bar{w}^l_a = 0\,, \\ p^a_{\bar{w}k} w^l_a = 0\,, \\ p^a_{\bar{w}k} \bar{w}^l_a = 0\,, \\ p^a_{wk} w^l_a = 0\,. \end{gathered} \tag{3.15}$$

The constraints (2.1), (2.4) and (3.14), (3.15) may be equivalently regarded as a system of conjugated second-class constraints and thus all subsequent Poisson bracket relations are in fact Dirac bracket relations on the surface defined by (2.1), (2.4) and (3.14), (3.15).

* (Anti-)symmetrization of indices is denoted as: $Y^{[ab]} \equiv \frac{1}{2}(Y^{ab} - Y^{ba})$, $Y^{(ab)} \equiv \frac{1}{2}(Y^{ab} + Y^{ba})$.

An important property of $S^{(\mathrm{I,II})}_{\text{harmonic}}$, eqs. (3.12), (3.13), is that the whole dynamics consists of pure gauge rotations. This is because the total harmonic hamiltonian is a linear combination of first-class constraints only and because there is no harmonic varible among v, u, w's to be identified with the evolution parameter. Thus, the new "harmonized" GS action (3.10) is physically equivalent to the standard one (3.1). In particular, the apparent breaking of invariance under arbitrary world sheet reparametrizations by $S^{(\mathrm{I,II})}_{\text{harmonic}}$ (3.12), (3.13) (where all harmonics are ξ-independent), does not have any physical effect.

Equipped with the harmonic formalism of the preceding section, we can now explicitly solve the problem posed in the introduction: the Lorentz-covariant and functionally independent separation of the first-class and second-class parts of the constraints D_A (3.6).

Let us consider first the type IIB case. The solution is provided by the following decomposition of the original MW constraints D_A^{α} with the help of the first generation harmonics $v_{\alpha}^{\pm\frac12}$, u_{μ}^{a}:

$$D_A^{\alpha} = \frac{1}{\Pi_A^{+}}\left(\sigma^{b} v^{-\frac12}\right)^{\alpha} D_{Ab}^{+\frac12} + \frac{1}{\Pi_A^{+}}\left(\not\Pi_A \sigma^{+} \sigma^{b} v^{-\frac12}\right)^{\alpha} G_{Ab}^{-\frac12} . \tag{3.16}$$

where $\Pi_A^{+}(\xi) = v^{-\frac12}\not\Pi_A(\xi) v^{+\frac12}$ is a *Lorentz scalar*.

By inverting (3.16) one obtains:

$$D_A^{+\frac12 a} = v^{+\frac12}\sigma^{a}\not\Pi_A D_A \tag{3.17}$$

(first-class generator of the κ-symmetry) and

$$G_A^{+\frac12 a} = \tfrac12\left(v^{-\frac12}\sigma^{a}\sigma^{+} D_A\right) \tag{3.18}$$

(second-class).

Now, the PB relations (3.7)–(3.9) are rewritten as follows:

$$\begin{aligned}
\left\{ D_A^{+\frac12 a}(\xi), D_B^{+\frac12 b}(\eta)\right\}_{\mathrm{PB}} = {} & -2i\delta_{AB} C^{ab}\Pi_A^{+}(\xi) T_A(\xi)\delta(\xi-\eta) \\
& -16(-1)^{A}\delta_{AB}\delta(\xi-\eta)\left(v^{+\frac12}\sigma^{(b}\theta_A'\right) D_A^{+\frac12 a)}(\xi) \\
& +8(-1)^{A}\delta_{AB}\delta(\xi-\eta) C^{ab}\left(v^{-\frac12}\sigma^{c}\theta_A'\right) D_{Ac}^{+\frac12}(\xi) \\
& +8\left(v^{-\frac12}\sigma^{a}\sigma^{c}\sigma^{d} v^{-\frac12}\right) G_{Ad}^{+\frac12}(\xi)\left[(-1)^{A}\delta_{AB}\delta'(\xi-\eta)\right] \\
& \times\left(v^{-\frac12}\sigma^{b}\sigma_{c}\sigma^{e} v^{-\frac12}\right) G_{Ae}^{+\frac12}(\eta),
\end{aligned} \tag{3.19}$$

$$\left\{ D_A^{-\frac{1}{2}a}(\xi), G_B^{+\frac{1}{2}b}(\eta) \right\}_{\mathrm{PB}} = 4(-1)^A \delta_{AB}\delta(\xi-\eta)\left(v^{-\frac{1}{2}}\sigma^b\sigma_c\sigma^+\theta_A'\right) \times \left(v^{+\frac{1}{2}}\sigma^a\sigma^c\sigma^d v^{-\frac{1}{2}}\right) G_{Ad}^{-\frac{1}{2}}(\xi), \tag{3.20}$$

$$\left\{ D_A^{+\frac{1}{2}a}(\xi), T_B(\eta) \right\}_{\mathrm{PB}} = 8(-1)^A \delta_{AB}\left[D_A^{+\frac{1}{2}a}(\xi)\delta'(\xi-\eta) + \tfrac{1}{2}\left(D_A^{+\frac{1}{2}a}(\xi)\right)'\delta(\xi-\eta)\right], \tag{3.21}$$

$$\left\{ G_A^{-\frac{1}{2},k}(\xi), T_B(\eta) \right\}_{\mathrm{PB}} = 4(-1)^A \delta_{AB}\left[G_A^{-\frac{1}{2},k}(\xi)\delta'(\xi-\eta) + \left(G_A^{-\frac{1}{2},k}(\xi)\right)'\delta(\xi-\eta)\right], \tag{3.22}$$

$$\left\{ T_A(\xi), T_B(\eta) \right\}_{\mathrm{PB}} = 8(-1)^A \delta_{AB}\left[T_A(\xi)\delta'(\xi-\eta) + \tfrac{1}{2}(T_A(\xi))'\delta(\xi-\eta)\right], \tag{3.23}$$

$$\left\{ G_A^{+\frac{1}{2}a}(\xi), G_B^{-\frac{1}{2}b}(\eta) \right\}_{\mathrm{PB}} = i\delta_{AB}C^{ab}\Pi_A^+(\xi)\delta(\xi-\eta). \tag{3.24}$$

Let us consider now the type IIA case. The irreducible covariant separation of D_A (3.6) into first-class and second-class parts reads now:

$$D_1^\alpha = \frac{1}{\Pi_1^+}\left(\sigma^b v^{+\frac{1}{2}}\right)^\alpha D_{1b}^{-\frac{1}{2}} + \frac{1}{\Pi_1^-}\left(\not{\Pi}_1\sigma^+\sigma^b v^{-\frac{1}{2}}\right)^\alpha G_{1b}^{-\frac{1}{2}}, \tag{3.25}$$

$$D_{2\alpha} = \frac{1}{\Pi_2^+}\left(\sigma^-\sigma^b v^{-\frac{1}{2}}\right)_\alpha D_{2b}^{+\frac{1}{2}} + \frac{1}{\Pi_2^-}\left(\not{\Pi}_2\sigma^b v^{+\frac{1}{2}}\right)_\alpha G_{2b}^{+\frac{1}{2}}, \tag{3.26}$$

or, inversely:

$$D_2^{+\frac{1}{2}a} = \tfrac{1}{2}\left(v^{-\frac{1}{2}}\sigma^a\sigma^+\not{\Pi}_2 D_2\right) \tag{3.27}$$

(first-class generator of the κ-symmetry),

$$G_2^{+\frac{1}{2}a} = v^{-\frac{1}{2}}\sigma^a D_2 \tag{3.28}$$

(second class), for $D_1^{+\frac{1}{2}a}$, $G_1^{+\frac{1}{2}a}$ the same as in (3.17), (3.18).

The PB relations become:

$$\begin{aligned}\left\{ D_2^{+\frac{1}{2}a}(\xi), D_2^{+\frac{1}{2}b}(\eta) \right\}_{\mathrm{PB}} = & -iC^{ab}\Pi_2^+(\xi)T_2(\xi)\delta(\xi-\eta) \\ & -4\delta(\xi-\eta)\left(v^{-\frac{1}{2}}\sigma^{(b}\sigma^+\theta_2'\right)D_2^{+\frac{1}{2}a)}(\xi) \\ & +4\delta(\xi-\eta)C^{ab}\left(v^{-\frac{1}{2}}\sigma^c\sigma^+\theta_2'\right)D_{2c}^{+\frac{1}{2}}(\xi) \\ & +2\left(v^{-\frac{1}{2}}\sigma^a\sigma_c\sigma^d v^{+\frac{1}{2}}\right)G_{2d}^{-\frac{1}{2}}(\xi)\delta'(\xi-\eta) \\ & \times\left(v^{-\frac{1}{2}}\sigma^b\sigma^c\sigma^e v^{+\frac{1}{2}}\right)G_{2e}^{+\frac{1}{2}}(\eta),\end{aligned} \tag{3.29}$$

$$\left\{ G_2^{+\frac{1}{2}a}(\xi), G_2^{+\frac{1}{2}b}(\eta) \right\}_{\mathrm{PB}} = 2iC^{ab}\Pi_2^+(\xi)\delta(\xi-\eta), \tag{3.30}$$

with the rest remaining the same as in (3.19)–(3.24).

4. Covariant gauge fixing conditions

We shall consider in detail in the case of type IIB GS superstrings the covariant gauge fixing procedure for the fermionic κ-invariance generated by $D^{+\frac{1}{2}a}$ (3.17) and for the reparametrization invariance generated by T_A (3.4). The corresponding results for type IIA GS superstrings are analogous and will not be explicitly listed.

It is convenient to perform the following change of variables which mixes in a non-trivial way the original superstring coordinates $X^{\mu}(\xi), \theta_A(\xi)$ and the harmonics.

$$X^{\mu}(\xi) \to X^{a}(\xi),\ X^{+}(\xi),$$

with

$$X^{a}(\xi) = u^{a}_{\mu} X^{\mu}(\xi), \tag{4.1}$$

$$X^{+}(\xi) = \left(v^{-\frac{1}{2}}\sigma_{\mu} v^{+\frac{1}{2}}\right) X^{\mu}(\xi),$$

$$\theta_{A\alpha} \to \phi^{a}_{A},\ \psi^{+\frac{1}{2}a}_{A}, \tag{4.2}$$

with

$$\theta_{A\alpha} = \frac{1}{2\sqrt{\Pi^{+}_{A}}}\left(\sigma^{+}\sigma^{b} v^{-\frac{1}{2}}\right)_{\alpha}\phi_{Ab} + \frac{1}{\Pi^{+}_{A}}\left(\not{\Pi}_{A}\sigma^{b} v^{+\frac{1}{2}}\right)_{\alpha}\psi^{+\frac{1}{2}}_{Ab} \tag{4.3}$$

and, correspondingly for the canonical momenta.

The new invariables are all *Lorentz scalars* by construction.

Eq. (4.3) allows to impose the *covariant* gauge fixing for $D^{+\frac{1}{2}a}_{A}$.

$$\psi^{+\frac{1}{2}a}_{A} \equiv \left(v^{+\frac{1}{2}}\sigma^{a}\theta_{A}\right) = 0. \tag{4.4}$$

After that, one has to compute the new Dirac brackets on the constraint surface. To achieve it, one uses the relations (3.19)–(3.24) and the PB relations:

$$\left\{ D^{-\frac{1}{2}a}_{A}(\xi), \psi^{+\frac{1}{2}b}(\eta)\right\}_{\mathrm{PB}} = -i\delta_{AB}C^{ab}\Pi^{+}_{A}(\xi)\delta(\xi-\eta),$$

$$\left\{ \psi^{+\frac{1}{2}a}_{A}(\xi), \psi^{-\frac{1}{2}b}_{B}(\eta)\right\}_{\mathrm{PB}} = 0,$$

$$\left\{ G^{-\frac{1}{2}a}_{A}(\xi), \psi^{+\frac{1}{2}b}_{B}(\eta)\right\}_{\mathrm{PB}} = 0, \tag{4.5}$$

The new Dirac brackets (DB) on the constraint surface

$$G^{+\frac{1}{2}}_{A} = 0, \qquad D^{+\frac{1}{2}a}_{A} = 0, \qquad \psi^{-\frac{1}{2}a}_{A} = 0$$

are (the vanishing ones are omitted):

$$\{\phi_A^a(\xi), \phi_B^b(\eta)\}_{\mathrm{DB}} = -i\delta_{AB}C^{ab}\delta(\xi-\eta), \tag{4.6}$$

$$\{\mathscr{P}^a(\xi), X^b(\eta)\}_{\mathrm{DB}} = -C^{ab}\delta(\xi-\eta),$$

$$\{\mathscr{P}^+(\xi), X^-(\eta)\}_{\mathrm{DB}} = \{\mathscr{P}^-(\xi), X^+(\eta)\}_{\mathrm{DB}} = \delta(\xi-\eta). \tag{4.7}$$

In order to expose the fact that actually the ϕ's represent according to (4.6) canonically conjugated pairs, one uses the second generation of harmonics (2.4):

$$\phi_A^k(\xi) = w_a^k\phi_A^a(\xi), \qquad \bar{\phi}_A^k(\xi) = w_a^{\bar{k}}\phi_A^a(\xi). \tag{4.8}$$

The Dirac brackets (4.6) are rewritten as:

$$\{\phi_A^k(\xi), \phi_B^l(\eta)\}_{\mathrm{DB}} = \{\bar{\phi}_A^k(\xi), \bar{\phi}_B^l(\eta)\}_{\mathrm{DB}} = 0,$$

$$\{\phi_A^k(\xi), \bar{\phi}_B^k(\eta)\}_{\mathrm{DB}} = -iC^{k\bar{k}}\delta_{AB}\delta(\xi-\eta). \tag{4.9}$$

Actually, in the type II theories, one can avoid the introduction of the second-generation harmonics by forming the holomorphic combinations (for the analog light-cone procedure, see ref. [1]):

$$\phi^a(\xi) = \sqrt{\tfrac{1}{2}}\,(\phi_1^a(\xi) + i\phi_2^a(\xi)), \qquad \bar{\phi}^a(\xi) = \sqrt{\tfrac{1}{2}}\,(\phi_1^a(\xi) - i\phi_2^a(\xi),$$

which form a canonically conjugated pair:

$$\{\phi^a(\xi), \phi^b(\xi)\}_{\mathrm{DB}} = 0,$$

$$\{\bar{\phi}^a(\xi), \bar{\phi}^b(\xi)\}_{\mathrm{DB}} = 0,$$

$$\{\phi^a(\xi), \bar{\phi}^b(\xi)\}_{\mathrm{DB}} = -iC^{ab}\delta(\xi-\eta).$$

However, we shall stick to the formalism with two generations of harmonics since it allows to treat on equal footing all types of superstrings, including those with only one θ-coordinate (e.g., the heterotic string).

The new small phase space is now formed by the variables:

$$X^{\pm}(\xi),\ X^a(\xi), \phi_A^k(\xi), u, v, w \tag{4.10}$$

and their respective canonical momenta:

$$\mathscr{P}^{\mp}(\xi), \mathscr{P}^a(\xi), \bar{\phi}_A^k(\xi),\ p_u, p_v, p_w. \tag{4.11}$$

The only variables here transforming nontrivially under the Lorentz group are $u_\mu^a, v_\alpha^{\pm\frac{1}{2}}$ and their momenta.

The reparametrization generator becomes*:

$$\tilde{T}_A(\xi) = T_A|_{G=D=\psi=0} = \left(\mathscr{P}(\xi) + (-1)^A X'(\xi)\right)^2$$

$$+2i(-1)^A\left[\phi_A^k(\xi)\bar{\phi}'_{Ak}(\xi) - \phi_A'^k(\xi)\bar{\phi}_{Ak}(\xi)\right]. \quad (4.12)$$

The expression does not explicitly contain the harmonic variables and it is bilinear.

Now we can further impose a *covariant* gauge fixing condition for the reparametrization invariance:

$$X^-(\xi) = q^+ + P^+\tau, \qquad \mathscr{P}^+(\xi) = \frac{1}{2\pi}P^+, \quad (4.13)$$

where q^+, P^+ are the corresponding zero modes in the mode expansions:

$$X^\pm(\xi) = q^\pm + \sum_{n\neq 0} X_n^\pm \mathrm{e}^{in\xi}, \qquad \mathscr{P}^\pm(\xi) = \frac{1}{2\pi}\left(P^\pm + \sum_{n\neq 0} P_n^\pm \mathrm{e}^{in\xi}\right)$$

and similarly for $X^a(\xi)$, $\mathscr{P}^a(\xi)$.

Eqs. (4.13) and:

$$\tilde{T}_A(\xi) = 0 \quad (4.14)$$

are solved in precisely the same way as the analogous equations in the noncovariant light-cone formulation [1].

Thus, on the surface defined by:

$$G_A^{+\frac{1}{2}a} = 0, \qquad D_A^{+\frac{1}{2}a} = 0, \qquad \psi_A^{+\frac{1}{2}a} = 0$$

and eqs. (4.13) (4.14), we obtain the following final results for the canonical variables, hamiltonian, gauge symmetries, and BRST charge.

The "small" phase space is parametrized by the canonical coordinates and momenta:

$$\left(q^-, X^a(\xi), \phi_A^k(\xi), u, v, w, P^+, \mathscr{P}^a(\xi), \bar{\phi}_A^k(\xi), p_u, p_v, p_w\right). \quad (4.15)$$

We stress once again that in the present harmonic formalism all light-cone *like* indices $\pm$, a are *internal* whereas *Lorentz-invariance* is manifest. (The Lorentz

* For any two Lorentz-vectors A_μ, B_μ we have in the harmonic basis: $A_\mu = u_\mu^a A_a - u_\mu^+ A^- - u_\mu^- A^+$ and similarly for B_μ, $A^\mu B_\mu = A^a B_a - A^+ B^- - A^- B^+$.

group acts only on the space-time indices of the first generation of harmonics, $u^a_\mu, v^{\pm\frac{1}{2}}_\alpha$ and their momenta.)

The canonical hamiltonian is given by:

$$H_{\mathrm{can}} = 2\pi \int_{-\pi}^{\pi} \mathrm{d}\xi \left[\mathscr{P}^a \mathscr{P}_a + X^{a\prime} X_a^{\prime} + i \sum_A (-1)^A \left(\phi^k_A \bar{\phi}^{\prime}_{Ak} - \phi^{k\prime}_A \bar{\phi}_{Ak} \right) \right]. \tag{4.16}$$

The only remaining constraints, after fixing κ and reparametrization are the harmonic first class constraints (2.7), (2.8), which take the following form on the small phase space (4.15):

$$\tilde{D}^{ab} = D^{ab} + \int_{-\pi}^{\pi} \mathrm{d}\xi \left[X^a(\xi) \frac{\delta}{\delta X_b(\xi)} - X^b(\xi) \frac{\delta}{\delta X_a(\xi)} \right],$$

$$\tilde{D}^{-+} = D^{-+} + q^+ \frac{\partial}{\partial q^+} - q^- \frac{\partial}{\partial q^-},$$

$$\tilde{D}^{+a} = D^{+a} + q^a \frac{\partial}{\partial q^-} + q^+ \frac{\partial}{\partial q_a} \tag{4.17}$$

(here q^a is the zero mode of $X^a(\xi)$ and q^+ now plays the role of a time, cf. eq. (4.13)).

$$\tilde{E}^{IJ} = E^{IJ} + \tfrac{1}{2} \sum_A \int_{-\pi}^{\pi} \mathrm{d}\xi\, \phi^k_A(\xi) (\rho^{IJ})_k^{\ l} \bar{\phi}_{Al}(\xi),$$

$$\tilde{E}^{+-} = E^{+-} + \tfrac{1}{2} \sum_A \int_{-\pi}^{\pi} \mathrm{d}\xi\, \phi^k_A(\xi) \bar{\phi}_{Ak}(\xi),$$

$$\tilde{E}^{+I} = E^{+I} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}} \sum_A \int_{-\pi}^{\pi} \mathrm{d}\xi\, \phi^k_A(\xi) (\rho^I)_{kl} \phi^l_A(\xi). \tag{4.18}$$

The final action of the "harmonized" GS superstring on the small phase space (4.15) takes now the form:

$$\begin{aligned} S_{\mathrm{GS}} = \int \mathrm{d}\tau \Bigg[& \int_{-\pi}^{\pi} \mathrm{d}\xi \left(\mathscr{P}^a\, \partial_\tau X_a + i \sum_A \bar{\phi}_{Ak}\, \partial_\tau \phi^k_A \right) \\ & - P^+ \partial_\tau q^- - H_{\mathrm{can}} + p^a_{u\mu}\, \partial_\tau u^\mu_a + p_v^{\mp\frac{1}{2}\alpha}\, \partial_\tau v_\alpha^{\pm\frac{1}{2}} \\ & - \Lambda_{ab} \tilde{d}^{ab} - \Lambda^{+-} \tilde{d}^{-+} - \Lambda^-_a \tilde{d}^{+a} + p^a_{wk}\, \partial_\tau w^k_a + p^a_{\bar{w}k}\, \partial_\tau \bar{w}^k_a \\ & - \lambda^{IJ} \tilde{e}^{IJ} - \lambda^{-+} \tilde{e}^{+-} - \lambda^{-I} \tilde{e}^{+I} \Bigg], \end{aligned} \tag{4.19}$$

where $\tilde{d}^{ab}, \ldots, \tilde{e}^{+I}$ are the classical counterparts of the constraints (4.17), (4.18) and H_{can} is given by (4.16).

The harmonized GS action (4.19) is explicitly Lorentz invariant (the indices $\pm$, a of P, q, X, $\mathscr{P}$ are *internal*, these variables being *Lorentz scalars* by construction).

The (first quantized) BRST charge corresponding to (4.19) reads:

$$Q^{\mathrm{BRST}}_{\mathrm{harmonic}} = Q^{(\mathrm{I})}_{\mathrm{harmonic}} + Q^{(\mathrm{II})}_{\mathrm{harmonic}}, \tag{4.20}$$

$$\begin{aligned} Q^{(\mathrm{I})}_{\mathrm{harmonic}} = {} & i\eta_{ab}\left[\tilde{D}^{ab} + \eta^{-a}\frac{\partial}{\partial\eta_b} - \eta^{-b}\frac{\partial}{\partial\eta_a} + \eta^a_d\frac{\partial}{\partial\eta_{bd}} - \eta^b_d\frac{\partial}{\partial\eta_{ad}}\right] \\ & + i\eta\left[\tilde{D}^{+-} - \eta^-_a\frac{\partial}{\partial\eta^-_a}\right] + i\eta^-_a\tilde{D}^{+a} \\ & + i\frac{\partial}{\partial\Lambda_{ab}}\frac{\partial}{\partial\tilde{\eta}^{ab}} + i\frac{\partial}{\partial\Lambda^{+-}}\frac{\partial}{\partial\tilde{\eta}} + i\frac{\partial}{\partial\Lambda^{+a}}\frac{\partial}{\partial\tilde{\eta}^+_a}, \end{aligned} \tag{4.21}$$

$$\begin{aligned} Q^{(\mathrm{II})}_{\mathrm{harmonic}} = {} & i\zeta^{IJ}\left[\tilde{E}^{IJ} + \zeta^{IK}\frac{\partial}{\partial\zeta^{JK}} - \zeta^{JK}\frac{\partial}{\partial\zeta^{IK}} + \zeta^{-I}\frac{\partial}{\partial\zeta^{-J}} - \zeta^{-J}\frac{\partial}{\partial\zeta^{-I}}\right] \\ & + i\zeta\left[\tilde{E}^{-+} - \zeta^{-I}\frac{\partial}{\partial\zeta^{-I}}\right] + i\zeta^{-I}\tilde{E}^{+I} \\ & + i\frac{\partial}{\partial\lambda^{IJ}}\frac{\partial}{\partial\tilde{\zeta}^{IJ}} + i\frac{\partial}{\partial\lambda^{+-}}\frac{\partial}{\partial\tilde{\zeta}} + i\frac{\partial}{\partial\lambda^{-I}}\frac{\partial}{\partial\tilde{\zeta}^{+I}}. \end{aligned} \tag{4.22}$$

The variables appearing in the above expression of the BRST charge are organized as follows:

$$\left[\begin{array}{cccc} \textit{Lagrange multiplier} & \textit{ghost} & \textit{antighost} & \textit{of the constraint} \\ \Lambda_{ab} & \eta_{ab} & \tilde{\eta}^{ab} & \tilde{D}^{ab} \\ \Lambda^{+-} & \eta & \tilde{\eta} & \tilde{D}^{-+} \\ \Lambda^{+a} & \eta^{-a} & \tilde{\eta}^{+a} & \tilde{D}^{+a} \\ \lambda^{IJ} & \zeta^{IJ} & \tilde{\zeta}^{IJ} & \tilde{E}^{IJ} \\ \lambda^{+-} & \zeta & \tilde{\zeta} & \tilde{E}^{-+} \\ \lambda^{-I} & \zeta^{-I} & \tilde{\zeta}^{+I} & \tilde{E}^{+I} \end{array}\right].$$

Since there are no normal ordering ambiguities in (4.17), (4.18) the constraints $\tilde{D}^{ab}, \ldots, \tilde{E}^{+I}$ form the same finite-dimensional Lie algebra as (2.7) and (2.8).

Therefore Q_{BRST} (4.20) is apparently anomaly free, the κ and the reparametrization invariances were solved explicitly and the Lorentz invariance is explicit and linearly realized.

Now, the question arises: where does the potential source of anomalies appear in the present harmonic superspace formalism? The answer is provided by looking at the consequences of the pure-gauge nature of the harmonic variables u, v, w for the covariantly gauge-fixed "harmonized" GS superstring (4.19).

As demonstrated in appendix B, the harmonic constraint equations (2.11) for the superstring wave function Φ in the central coordinate basis $(X^{\mu}(\cdot), \theta_{A\alpha}(\cdot); u, v, w)$ before any gauge-fixing procedure imply as trivial consequences the eqs. (B.9)–(B.10):

$$D^{-a}\Phi = 0, \qquad E^{-I}\Phi = 0. \tag{4.23}$$

Here the harmonic differential operators D^{-a}, E^{-I} are defined as:

$$D^{-a} = u_{\mu}^{-} \frac{\partial}{\partial u_{\mu a}} + \tfrac{1}{2} v^{+\frac{1}{2}} \sigma^{-} \sigma^{a} \frac{\partial}{\partial v^{+\frac{1}{2}}},$$

$$E^{-I} = \sqrt{\tfrac{1}{2}}\, \overline{w}_{a}^{k} (\tilde{\rho}^{I})_{k}^{\ l} \frac{\partial}{\partial w_{a}^{l}}, \tag{4.24}$$

and possess a simple geometrical meaning explained in appendix B.

On the small phase space (4.15), the operators (4.24) acquire the form (cf. eqs. (4.17), (4.18)):

$$\tilde{D}^{-a} = D^{-a} + \int_{-\pi}^{\pi} \mathrm{d}\xi \Bigg\{ X^{-}(\xi) \frac{\delta}{\delta X_{a}(\xi)} - iX^{a}(\xi) \mathscr{P}^{-}(\xi)$$

$$+ \frac{1}{4P^{+}} \Gamma^{abcd}(u, v) \sum_{A} \left(\mathscr{P}_{b} + (-1)^{A} X_{b} \right)$$

$$\times \left[\overline{w}_{ck} \phi_{A}^{k} + w_{c\dot{k}} \phi_{A}^{\dot{k}} \right] \left[\overline{w}_{dl} \phi_{A}^{l} + w_{d\dot{l}} \phi_{A}^{\dot{l}} \right] \Bigg\}, \tag{4.25}$$

$$\tilde{E}^{-I} = E^{-I} - \tfrac{1}{2} \sqrt{\tfrac{1}{2}} \sum_{A} \int_{-\pi}^{\pi} \mathrm{d}\xi\, \overline{\phi}_{A}^{\dot{k}} \tilde{\rho}_{\dot{k}\dot{l}}^{I} \overline{\phi}_{A}^{\dot{l}}. \tag{4.26}$$

In eq. (4.25) the following notations are used:

$$\mathscr{P}_a(\xi) = -i\frac{\delta}{\delta X^a(\xi)},$$

$$\Gamma^{abcd}(u,v) \equiv v^{-\frac{1}{2}}\sigma^{[a}\sigma^{b}\sigma^{c}\sigma^{d]}\sigma^{+}v^{-\frac{1}{2}} + C^{bc}C^{ad} - C^{bd}C^{ac},$$

$$X^{-}(\xi) = q^{-} - \frac{i}{2\pi P^{+}}\sum_{n\neq 0}\frac{e^{in\xi}}{n}\int_{-\pi}^{\pi}\mathrm{d}\eta\, e^{-in\eta}$$

$$\times\left[\mathscr{P}^a(\xi)X_a'(\xi) + \tfrac{1}{2}i\sum_A\left(\phi_A^k\bar{\phi}_{Ak}' - \phi_A^{k\prime}\bar{\phi}_{Ak}\right)\right], \tag{4.27}$$

$$\mathscr{P}^-(\xi) = \frac{2\pi}{P^+}\left[\mathscr{P}^a(\xi)\mathscr{P}_a(\xi) + X'^a(\xi)X_a'(\xi) + i\sum_A(-1)^A\left(\phi_A^k\bar{\phi}_{Ak}' - \phi_A^{k\prime}\bar{\phi}_{Ak}\right)\right]. \tag{4.28}$$

Thus, for the covariantly gauge-fixed superstring action (4.19), the consequences (4.24) of the pure gauge nature of the harmonics u, v, w, i.e. eqs. (2.11), imply:

$$\tilde{D}^{-a}\Phi\left[q^+, q^-, X^a(\cdot), \phi_a^k(\cdot); u, v, w\right) = 0, \tag{4.29}$$

$$\tilde{E}^{-I}\Phi\left[q^+, q^-, X^a(\cdot), \phi_a^k(\cdot); u, v, w\right) = 0. \tag{4.30}$$

Let us particularly emphasize that eqs. (4.29), (4.30) *do not* correspond to additional (quantized) first-class harmonic constraints $\tilde{D}^{-a}$, $\tilde{E}^{-I}$ (4.17), (4.18), but rather are *consequences* of the harmonic constraint equations:

$$\tilde{D}^{ab}\Phi = 0, \ldots, \qquad \tilde{E}^{+I}\Phi = 0 \tag{4.31}$$

due to the specific form of the harmonic expansions in the central coordinate basis (B.1), (B.6).

Now, the source of potential anomalies in the harmonic superspace formalism is revealed in the consistency condition for eqs. (4.29):

$$[\tilde{D}^{-a}, \tilde{D}^{-b}] = 0. \tag{4.32}$$

Inspecting formula (4.25) together with (4.27), (4.28) we see that due to the presence of trilinear terms in (4.25) there are nontrivial cancellations leading to the result (4.32). These cancellations resemble completely the nontrivial cancellations in the commutator of the $D = 10$ coset Lorentz generators $[J^{-a}, J^{-b}]$ in the noncovariant light-cone formalism (ch. 5 of ref. [1]).

Thus, we conclude that possible quantum anomalies in $[\tilde{D}^{\,a}, \tilde{D}^{-b}]$ in (4.32) for $D \neq 10$ infringe the pure gauge property of the harmonic variables u, v, w.

5. Super-Poincaré algebra

In the present formalism all Lorentz indices are absorbed by the first-generation harmonics $v_\alpha^{+\frac{1}{2}}, u_\mu^a$. Therefore, the Poincaré algebra acts in a manifestly covariant form in the "harmonized" GS superstring theory (4.19).

The explicit form of the Poincaré generators reads:

$$M^{\mu\nu} = u^{a\mu}\frac{\partial}{\partial u_\nu^a} - u^{a\nu}\frac{\partial}{\partial u_\mu^a} - \frac{1}{2}\left(v^{+\frac{1}{2}}\sigma^{\mu\nu}\frac{\partial}{\partial v^{+\frac{1}{2}}} + v^{-\frac{1}{2}}\sigma^{\mu\nu}\frac{\partial}{\partial v^{-\frac{1}{2}}}\right), \tag{5.1}$$

$$P^\mu = u_a^\mu P^a - u^{-\mu}P^+ - \frac{1}{P^+}u^{+\mu}H_{\mathrm{can}}, \tag{5.2}$$

where $(\sigma_{\mu\nu})_\beta^\alpha = \sigma_{[\mu}^{\alpha\gamma}\sigma_{\nu]\gamma\beta}$. With the risk of being surreptitious let us remind the reader that P^+ is a Lorentz scalar and the superscript + refers to its charge under the SO(1,1) internal gauge group. Its appearance in various expressions is perfectly consistent with the *explicit Lorentz-invariance* of our formulation.

Let us now consider the supersymmetry (SUSY) transformations (we shall explicitly treat the case of type IIB theory, the case IIA being completely analogous). The original GS action (3.1) is invariant under:

$$\delta_{\mathrm{SS}}X^\mu(\xi) = -i\sum_A \varepsilon_A\sigma^\mu\theta_A(\xi), \tag{5.3}$$

$$\delta_{\mathrm{SS}}\theta_{A\alpha}(\xi) = \varepsilon_{A\alpha}. \tag{5.4}$$

In the hamiltonian framework the corresponding SUSY generators read:

$$Q_A^\alpha = \int_{\pi}^{\pi}\mathrm{d}\xi\, Q_A^\alpha(\xi), \tag{5.5}$$

$$Q_A^\alpha(\xi) = -ip_{\theta A}^\alpha + \left[\mathscr{P}^\mu + (-1)^A(X'^\mu + i\theta_A\sigma^\mu\theta_A')\right](\sigma_\mu\theta_A)^\alpha. \tag{5.6}$$

One can verify straightforwardly that Q_A^α commutes with all superstring constraints $T_A(\xi)$, $D_A(\xi)$ (eqs. (3.4), (3.6)). Also, by definition, all the harmonics u, v, w are inert under SUSY.

However, in order to preserve our covariant gauge fixing condition of the κ-symmetry (4.4), we have to supplement the SUSY generators (5.6) by an appropriate κ-symmetry rotation:

$$\bar{\delta}_{\mathrm{SS}} = \sum_A \varepsilon_{A\alpha}Q_A^\alpha - \sum_A\int_{-\pi}^{\pi}\mathrm{d}\xi\,\frac{1}{\Pi_A^+(\xi)}\left(\varepsilon_A\sigma^a v^{+\frac{1}{2}}\right)D_{A\dot{a}}^{+\frac{1}{2}}(\xi) \equiv \sum_A \varepsilon_{A\alpha}\bar{Q}_A^\alpha, \tag{5.7}$$

$$\left\{\bar{Q}_A^\alpha, \psi_B^{+\frac{1}{2}a}(\xi)\right\}_{\mathrm{PB}} = 0.$$

Evaluating $\overline{Q}^{\alpha}_{A}$ (eq. (5.7)) on the small phase space (4.15) we obtain:

$$\overline{Q}^{\alpha}_{A} = -\sqrt{\frac{2P^{+}}{\pi}}\left(\sigma^{a}v^{-\frac{1}{2}}\right)^{\alpha}\int_{-\pi}^{\pi}\mathrm{d}\xi\left[\overline{w}^{a}_{k}\phi^{k}_{A}+w^{a}_{k}\overline{\phi}^{k}_{A}\right]$$

$$-\sqrt{\frac{2\pi}{P^{+}}}\left(\sigma^{+}\sigma_{b}\sigma_{a}v^{-\frac{1}{2}}\right)^{\alpha}\int_{-\pi}^{\pi}\mathrm{d}\xi\left[\mathscr{P}_{b}+(-1)^{A}X'_{b}\right]\left[\overline{w}^{a}_{k}\phi^{k}_{A}+w^{a}_{k}\overline{\phi}^{k}_{A}\right]. \quad (5.8)$$

Note that normal ordering ambiguities do not arise in the expression (5.8).

Now, using (5.1), (5.2), (5.8) one obtains the commutation relations:

$$\left\{\overline{Q}^{\alpha}_{A},\overline{Q}^{\beta}_{B}\right\}=2\delta_{AB}\not{P}^{\alpha\beta},$$

$$\left[\overline{Q}^{\alpha}_{A},P^{\mu}\right]=0,$$

$$\left[M^{\mu\nu},\overline{Q}^{\alpha}_{A}\right]=\tfrac{1}{2}\left(\sigma^{\mu\nu}\right)^{\alpha}_{\beta}\overline{Q}^{\beta}_{A}. \quad (5.9)$$

Hence, the super-Poincaré algebra of the "harmonized" GS superstring (4.19) is anomaly free upon quantization.

The explicit form of the SUSY transformations generated by (5.8) on the canonical variables which enter (4.19) is:

$$\bar{\delta}_{\mathrm{SS}}X^{a}(\xi)=i\sqrt{\frac{2\pi}{P^{-}}}\sum_{A}\left(\varepsilon_{A}\sigma^{+}\sigma^{a}\sigma_{b}v^{-\frac{1}{2}}\right)\left[\overline{w}^{b}_{k}\phi^{k}_{A}(\xi)+w^{b}_{k}\overline{\phi}^{k}_{A}(\xi)\right] \quad (5.10)$$

$$\bar{\delta}_{\mathrm{SS}}q = \frac{-i}{\sqrt{2\pi P^{+}}}\sum_{A}\left(\varepsilon_{A}\sigma_{a}v^{-\frac{1}{2}}\right)\int_{-\pi}^{\pi}\mathrm{d}\xi\left[\overline{w}^{a}_{k}\phi^{k}_{A}+w^{a}_{k}\overline{\phi}^{k}_{A}\right]$$

$$-i\sqrt{\tfrac{1}{2}\pi}\,(P^{+})^{-3/2}\sum_{A}\left(\varepsilon_{A}\sigma^{-}\sigma_{a}\sigma_{b}v^{-\frac{1}{2}}\right)$$

$$\times\int_{-\pi}^{\pi}\mathrm{d}\xi\left[\mathscr{P}^{a}(\xi)+(-1)^{A}X'^{a}(\xi)\right]\left[\overline{w}^{b}_{k}\phi^{k}_{A}(\xi)+w^{b}_{k}\overline{\phi}^{k}_{A}(\xi)\right], \quad (5.11)$$

$$\bar{\delta}_{\mathrm{SS}}\mathscr{P}^{a}(\xi)=\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\bar{\delta}_{\mathrm{SS}}X^{a}(\xi)\right), \quad (5.12)$$

$$\bar{\delta}_{\mathrm{SS}}P^{-}=0, \quad (5.13)$$

$$\bar{\delta}_{\mathrm{SS}}\phi^{k}_{A}(\xi)=-\sqrt{\frac{2P^{+}}{\pi}}\,w^{k}_{a}\left(v^{-\frac{1}{2}}\sigma^{a}\varepsilon_{A}\right)-\sqrt{\frac{2\pi}{P^{+}}}\,w^{k}_{a}\left(v^{-\frac{1}{2}}\sigma^{a}\sigma^{b}\sigma^{-}\varepsilon_{A}\right)$$

$$\times\left[\mathscr{P}_{b}(\xi)+(-1)^{A}X'_{b}(\xi)\right] \quad (5.14)$$

and similarly for $\bar{\delta}_{\mathrm{SS}}\overline{\phi}^{k}_{A}$ with $w^{k}_{a}\to\overline{w}^{k}_{a}$.

The SUSY transformations (5.10)–(5.14) acts in a nonstandard way in as far as they mix canonical coordinates and their conjugate momenta. Also, for q^- the SUSY transformation even acts nonlinearly (5.11). Let us stress, however that the super-Poincaré algebra (5.1), (5.2), (5.8), (5.9) is realized in an explicitly Lorentz-covariant form: the SUSY parameters $\varepsilon_{A\alpha}$ are ordinary constant $D=10$ MW spinors.

6. Conclusions

Using ideas from our previous works [11, 13], we succeeded in the present paper to reformulate the GS superstring theory as a constraint system possessing Lorentz-covariant and functionally independent constraints.

The crucial step in our formalism was the introduction of additional (pure gauge) bosonic degrees of freedom – Lorentz-spinor and Lorentz-vector harmonics corresponding to the homogeneous space $SO(1,9)/(SO(8)\times SO(1,1))$ – which enabled us to perform covariant and functionally independent disentangling of the first-class and second-class parts of the superstring's fermionic constraints. These harmonics were also used to impose Lorentz-covariant gauge-fixing conditions for the local fermionic κ-invariance and for the reparametrization invariance.

While for type II theories the Lorentz spinor/vector harmonics are sufficient, for heterotic-type superstrings we had to introduce a second generation of harmonic degrees of freedom realizing the coset space $SO(8)/(SU(4)\times U(1))$. These are necessary in order to disentangle the resulting Dirac brackets for the remaining canonical superstring coordinates.

Our results can be summarized by the following Lorentz covariant formulae:

(1) The list of phase space variables (4.15).

(2) The canonical hamiltonian (4.16) in which the reparametrization and κ gauge-symmetries are fixed.

(3) The action (4.19).

(4) The list of first class constraints (4.17)–(4.18) left in (4.19) after fixing the reparametrization and κ-invariances. They form a finite-dimensional Lie algebra corresponding to the purely harmonic constraints.

(5) The $Q^{\mathrm{BRST}}_{\mathrm{harmonic}}$ (eq. (4.20)) which carries information only about the harmonic invariances since the others have been fixed.

The super-Poincaré algebra acts in a manifestly Lorentz-covariant way, but some translations and SUSY transformations are nonlinearly realized. An important feature of our formalism is that the super-Poincaré algebra is apparently free of quantum anomalies. The potential anomalies in the covariant harmonic superspace approach manifest themselves in the possible (for $D\neq 10$) breakdown of the pure gauge nature of the auxiliary harmonic variables.

Given the BRST charge (4.20) one can write down the superstring field action along the lines of refs. [20, 16].

However, in order to keep super-Poincaré invariance linearly realized, there exists another option for further investigation. Namely, one can use the generalized BFV formalism of ref. [21], to convert the covariant irreducible second-class constraints of the GS superstring into first-class ones at the expense of introducing additional dynamical degrees of freedom. This task is now under study.

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Appendix A

SPINOR NOTATIONS AND CONVENTIONS

The $D = 10$ γ-matrices and $D = 10$ charge conjugation matrix are taken in the following representation:

$$\Gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_\alpha^{\dot\beta} \\ (\tilde\sigma^\mu)_{\dot\alpha}^{\beta} & 0 \end{pmatrix},$$

$$C_{10} = \begin{pmatrix} 0 & C^{\alpha\dot\beta} \\ (-C)^{\dot\alpha\beta} & 0 \end{pmatrix},$$

$$\Gamma^{11} \equiv \Gamma^0\Gamma^1 \dots \Gamma^9 = \begin{pmatrix} \delta_\alpha^\beta & 0 \\ 0 & -\delta_{\dot\alpha}^{\dot\beta} \end{pmatrix}.$$

Indices of $D = 10$ left- (right-) handed MW spinors ϕ_α, $\psi_{\dot\alpha}$ are raised by means of C_{10}:

$$\phi^{\dot\alpha} = (-C)^{\dot\alpha\beta}\phi_\beta,$$

$$\psi^\alpha = C^{\alpha\dot\beta}\psi_{\dot\beta}.$$

Throughout the paper we use $D=10$ σ-matrices with undotted indices only:

$$(\sigma^{\mu})^{\alpha\beta}=C^{\alpha\dot{\alpha}}(\tilde{\sigma}^{\mu})_{\dot{\alpha}}^{\ \beta},$$

$$(\sigma^{\mu})_{\alpha\beta}=(-C)^{-1}_{\beta\dot{\beta}}(\sigma^{\mu})_{\alpha}^{\ \dot{\beta}},$$

$$(\sigma^{\mu})_{\alpha\gamma}(\sigma^{\nu})^{\gamma\beta}+(\sigma^{\nu})_{\alpha\gamma}(\sigma^{\mu})^{\gamma\beta}=-2\delta_{\alpha}^{\beta}\eta^{\mu\nu},$$

$$\eta_{\mu\nu}=\mathrm{diag}(-,+,\dots,+).$$

For the $D=8$ γ-matrices and $D=8$ charge conjugation matrix we use the following representation:

$$\Gamma_8^{i}=\begin{pmatrix}0 & (\gamma^{i})_{a}^{\ \dot{b}}\\ (\tilde{\gamma}^{i})_{\dot{a}}^{\ b} & 0\end{pmatrix},$$

$$C_8=\begin{pmatrix}C^{ab} & 0\\ 0 & (-C)^{\dot{a}\dot{b}}\end{pmatrix},$$

$$C^{ab}=C^{ba}.$$

Indices of SO(8) (s) and (c) spinors $\phi_a, \psi_{\dot{a}}$ are raised as:

$$\phi^{a}=C^{ab}\phi_b,\ \psi^{\dot{a}}=(-C)^{\dot{a}\dot{b}}\psi_{\dot{b}}.$$

The $D=6$ γ-matrices and $D=6$ charge conjugation matrix are taken in the representation:

$$\Gamma_6^{I}=\begin{pmatrix}0 & (\rho^{I})_{k}^{\ \dot{l}}\\ (\tilde{\rho}^{I})_{\dot{k}}^{\ l} & 0\end{pmatrix},$$

$$C_6=\begin{pmatrix}0 & C^{k\dot{l}}\\ (C)^{\dot{k}l} & 0\end{pmatrix},$$

$$C^{k\dot{l}}=C^{\dot{l}k}.$$

Indices of SO(6), i.e. SU(4), (4) and $(\bar{4})$ spinors $\phi^{k}, \bar{\psi}^{\dot{k}}$ are lowered by means of C_6^{-1}:

$$\phi_{\dot{k}}=C_{\dot{k}l}\phi^{l},$$

$$\bar{\psi}_{k}=C_{k\dot{l}}\bar{\phi}^{\dot{l}}.$$

In particular, $D=6$ σ-matrices with undotted indices are antisymmetric:

$$\rho^I_{kl} = (\rho^I)^i_k C_{il} = -\rho^I_{lk}\,.$$

Appendix B

ALGEBRAIC PROPERTIES OF THE HARMONIC EXPANSION

Here we shall consider only the case of harmonic expansion with respect to the second generation harmonics (2.4) realizing the compact coset space SO(8)/(SU(4) × U(1)). The case of harmonic expansion with respect to the first generation harmonics (2.1) was already discussed in [13]. The general form of the SO(8)/(SU(4) × U(1)) harmonic expansion of an arbitrary super(string)-field reads:

$$\Phi[X^\mu(\cdot),\theta_A(\cdot),u,v,w] = \sum_{n=0}^{\infty}\frac{1}{n!}w^2_{a_1b_1}\dots w^2_{a_nb_n}$$
$$\times\Phi_n^{[a_1b_1]\dots[a_nb_n]}[X^\mu(\cdot),\theta_A(\cdot),u,v]\,, \qquad \text{(B.1)}$$

where:

$$w^2_{ab} \equiv w^k_{[a}\bar{w}_{b]k} \qquad \text{(B.2)}$$

is the only independent SU(4) × U(1)-invariant combination of w's (2.4), and the coefficient super(string)-fields Φ_n in (B.1) are traceless with respect to any pair of SO(8) indices. The last restriction on Φ_n is due to the identity:

$$w^2_{ac}w^{2cb} = \tfrac{1}{2}\delta^b_a\,.$$

The expansion (B.1) already reflects the fact that:

$$E^{IJ}\Phi = 0\,,\qquad E^{--}\Phi = 0\,, \qquad \text{(B.3)}$$

where E^{IJ}, E^{-+} are defined in (2.8). Let us stress that, as it is the case in all harmonic expansions of superfields [17], the coefficient fields Φ_n in (B.1) carry indices belonging to the "great" group SO(8), but are singlets with respect to the small group SU(4) × U(1).

Now, using the simple identity:

$$E^{+I}w^2_{ab} = -w^k_a\rho^I_{kl}w^l_b$$

it is straightforward to show that the harmonic equation

$$E^{+I}\Phi[X^\mu(\cdot),\theta_A(\cdot);u,v,w] = 0\,, \qquad \text{(B.4)}$$

where Φ is of the form (B.1), is solved as:

$$\Phi_n = 0, n \geqslant 1; \qquad \Phi_0 = \Phi_0[X^\mu(\cdot), \theta_A(\cdot),; u, v] = \text{arbitrary}. \tag{B.5}$$

Thus, we have shown that any harmonic super(string)-field Φ satisfying (B.3), (B.4) is in fact independent of the harmonics w_a^k, $\bar{w}_a^k$.

Next, consider the only surviving coefficient field Φ_0 of the original field Φ whose harmoic expansion reads:

$$\Phi_0(X^\mu(\cdot), \theta_A(\cdot); u, v) = \sum_{r,s=0}^{\infty} \left[u_{\mu_1}^{a_1} \ldots u_{\mu_r}^{a_r}\right]_{\text{singlet part in}(a_1 \ldots a_r)}$$
$$\times v_{\alpha_1}^{+\frac{1}{2}} \ldots v_{\alpha_s}^{+\frac{1}{2}} v_{\alpha_{s+1}}^{-\frac{1}{2}} \ldots v_{\alpha_{2s}}^{-\frac{1}{2}} \Phi_{0,rs}^{\mu_1 \ldots \mu_r \alpha_1 \ldots \alpha_{2s}}[X^\mu(\cdot), \theta_A(\cdot)], \tag{B.6}$$

where the coefficient fields are subject to the tracelessness conditions explained in ref. [13].

If (B.6) is subject to the equations:

$$D^{ab}\Phi_0 = 0, \qquad D^{-+}\Phi_0 = 0, \qquad D^{+a}\Phi_0 = 0, \tag{B.7}$$

where D^{ab}, D^{-+}, D^{+a} are defined in (2.7), we get, as already demonstrated in [13], that Φ (B.1) is independent of the first generation harmonics u_μ^a, $v_\alpha^{\pm\frac{1}{2}}$ too:

$$\Phi_{0,rs}^{\mu_1 \ldots \mu_r \alpha_1 \ldots \alpha_{2s}} = 0, \qquad (r,s) \neq (0,0), \qquad \Phi_{0,00}[X^\mu(\cdot), \theta_A(\cdot)] = \text{arbitrary}. \tag{B.8}$$

Now, from (B.5), (B.8) we obtain as a trivial consequence of eqs. (B.3), (B.4), (B.7), that also the following equations are satisfied:

$$E^{-I}\Phi \equiv \left[-\frac{1}{\sqrt{2}} \bar{w}_a^k (\tilde{\rho}^I)_k^{\ l} \frac{\partial}{\partial w_a^l}\right]\Phi = 0, \tag{B.9}$$

$$D^{-a}\Phi \equiv \left[u_\mu \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2} v^{+\frac{1}{2}} \sigma^- \sigma^a \frac{\partial}{\partial v^{+\frac{1}{2}}}\right]\Phi = 0. \tag{B.10}$$

The new harmonic differential operators introduced in (B.9), (B.10) have simple geometrical meaning. E^{-I} is precisely the second half of the coset generators corresponding to SO(8)/(SU(4) × U(1)) and thus E^{IJ}, E^{+-}, E^{+I}, E^{-I} span the whole SO(8) algebra. Similarly, D^{-a} is identified as the second half of the coset generators corresponding to SO(1,9)/(SO(8) × SO(1,1)) such that the operators $\mathscr{D}^{ab} \equiv D^{ab} + \frac{1}{2}(v^{+\frac{1}{2}}\sigma^{ab}\partial/\partial v^{+\frac{1}{2}} + v^{-\frac{1}{2}}\sigma^{ab}\partial/\partial v^{-\frac{1}{2}})$, D^{-+}, D^{+a}, D^{-a} span the whole

SO(1,9) algebra (in the absence of the contribution from the second generation harmonics w to D^{ab} (2.7)).

Let us stress once again that E^{-I} (B.9), D^{-a} (B.10) are *not independent* (quantized) first class constraints. They are *not in addition* to E^{+-}, E^{+I}, E^{IJ}, D^{+-}, D^{+a}, D^{ab}, *but* eqs. (B.9), (B.10) are trivial *consequences* of (B.3), (B.4), (B.7) and of the specific form of the harmonic expansions (B.1), (B.6).

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